**NON-PARAMETRIC TEST**

**Mann-Whitney U-Test**

Mann-Whitney U-Test is a most powerful non-parametric test. It is used to test whether the two independent random samples drawn from population with unknown medians are same or not.

**Case-I:** Small sample case when and

**Problem:** To test,

**Null hypothesis (H0):** F (X) = F(Y) ⇒(y); there is no significance difference between median of X and median of Y i.e. both sample values come from similar population.

**Alternative hypothesis H1:** F (X) F(Y) ⇒; Two-tailed test

**H1:** F (X) >F (Y) ⇒; One-tailed test

**H1:** F (X) <F (Y) ⇒; One-tailed test

**Test statistic:** Under H0, test statistic is

U0 = Minimum of (U1 or U2)

Where,

U1 = n1n2 +

U2 = n1n2 + = n1n2 – U1

R1 = Sum of rank of first sample

R2 = Sum of rank of second sample

Note: Rank obtained by combined both samples

**Critical region:** Next for a pre-assigned level of significance. We from the Mann-Whitney U-table table the critical value is p0 = p **()**.

**Alternatively:** Next for a pre-assigned level of significance and (n1, n2). We from the Mann-Whitney U-table table the critical value is for one tailed and for two tailed test.

**Decision:** If for one tailed and; we reject H0. Otherwise accept H0.

**Alternatively:** Since; we reject H0. Otherwise accept H0.

**Case-II:** Large sample case when n1 10 and n2 > 10.

In case of large sample the sampling distribution of **U** is approximately normal with mean and variance

**Test statistic:** Under H0, test statistics is given by

Z=

Where,

=

=; if tied occurs within the samples.

=; if tied occurs between the samples.

T =

= No. of times rank repeated between the samples.

**Critical region:** Next for a pre-assigned level of significance, the probability (p0) is associated with the values as extreme as z, we obtained from z- table is

p0 = p ().

**Decision:** For one-tailed test, if p0 , we reject H0. Otherwise reject H0.

For two-tailed test, if 2 p0 , we reject H0. Otherwise reject H0.

**Example (1):** Two groups of rats, one group consisting of trained one another group untrained one (i.e. controlled) have the following number of trials to achieve certain criterion:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Trained rats | 78 | 64 | 75 | 45 | 82 |
| Untrained rats | 110 | 70 | 53 | 51 |  |

Use Mann-Whitney U-Test if there is a difference between the two average numbers of trials of trained and untrained rats. OR Test whether the distribution of number of trials of two groups of rats are identical or not?

**Solution:** Given,

n1 = 5, n2 = 4

|  |  |  |  |
| --- | --- | --- | --- |
| Trained | Untrained | Rank of Trained | Rank of untrained |
| 78 | 110 | 7 | 9 |
| 64 | 70 | 4 | 5 |
| 75 | 53 | 6 | 3 |
| 45 | 51 | 1 | 2 |
| 82 |  | 8 |  |
|  |  | = 26 | = 19 |

U1 = n1n2 + = 54 + = 20 + = 9

U2 = n1n2 + = 54 + = 20 + = 11

Now,

**Problem:** To test,

**Null hypothesis (H0):** F (X) = F(Y) ⇒(y); there is no significance difference between the two average numbers of trials of trained and untrained rats.

**Alternative hypothesis H1:** F (X) F(Y) ⇒; Two-tailed test

**Test statistic:** Under H0, test statistic is

U0 = Minimum of (U1 or U2) = 9

**Critical region:** Next for a pre-assigned level of significance = 0.05 and (n1 = 5, n2 = 4). We from the Mann-Whitney U-table table the critical value is

p0 = p **()** = p **() =** 0.4524

= 2 0.4524 = 0.9048

**Decision:** Since 2; we accept H0.

**Conclusion:**  There is no significance difference between the two average numbers of trials of trained and untrained rats.

**Example (2):** Two types of paints are to be tested. Type-I is somewhat cheaper than Type-II. The test consists in giving scores to the paints after they have been exposed to certain whether conditions for a period of 6 months. Eight samples of paint type-I and seven samples of paint Type-II scored as follows:

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Paint | Scores | | | | | | | |
| Type-I | 12 | 15 | 8 | 10 | 10 | 6 | 20 | 11 |
| Type-II | 16 | 25 | 18 | 22 | 15 | 22 | 18 |  |

We should like to adopt type-I the cheaper one unless we have definite reason to believe that type-II better. Test at 5% level of significance by using Mann-Whitney U-Test.

**Solution:** Given,

n1 = 8, n2 = 7

|  |  |  |  |
| --- | --- | --- | --- |
| Type-I | Type-II | Rank of type-I | Rank of type-II |
| 12 | 16 | 6 | 9 |
| 15 | 25 | 7.5 | 15 |
| 8 | 18 | 2 | 10.5 |
| 10 | 22 | 3.5 | 13.5 |
| 10 | 15 | 3.5 | 7.5 |
| 6 | 22 | 1 | 13.5 |
| 20 | 18 | 12 | 10.5 |
| 11 |  | 5 |  |
|  |  | = 40.5 | = 79.5 |

U1 = n1n2 + = 87 + = 56 + = 51.5

U2 = n1n2 + = 87 + = 56 + = 4.5

Now,

**Problem:** To test,

**Null hypothesis (H0):** F (X) = F(Y) ⇒(y); there is no significance difference between medians of type-I and type-II.

**Alternative hypothesis H1:** F (X) < F(Y) ⇒; one-tailed test

**Test statistic:** Under H0, test statistic is

U0 = Minimum of (U1 or U2) = 4.5

**Critical region:** Next for a pre-assigned level of significance = 0.05 and (n1 = 8, n2 = 7). We from the Mann-Whitney U-table table the critical value U is = = 10

**Decision:** Since U<; we reject H0.

**Conclusion:**  There is significance difference between the medians of type-I and type-II paints. That is type-II paint is better than type-I paint.

**Example (3):** The amount of rainfall (in mm) at two districts Dhanusha and Sarlahi as revealed by the record of Meteorological Division, during monsoon were as follows:

|  |  |
| --- | --- |
| Dhanusha | 15, 10, 14, 15, 20, 8, 17, 22, 12, 9 |
| Sarlahi | 12, 7, 11, 19, 23, 4, 8, 3, 6, 10, 13, 6, 9, 12, 7, 10, 21, 4, 9, 6, 11, 16, 14 |

Do the data provide sufficient evidence to conclude that amount of rainfall at Dhanusha is significantly lower than Sarlahi? Analyze the data at = 0.05 level of significance.

**Solution:** Given,

n1 = 10, n2 = 23 n = n1 + n2 = 10 + 23 = 33

|  |  |  |  |
| --- | --- | --- | --- |
| Dhanusha | Sarlahi | Rank of Dhanusha | Rank of Sarlahi |
| 15 | 12 | 25.5 | 20 |
| 10 | 7 | 15 | 7.5 |
| 14 | 11 | 23.5 | 17.5 |
| 15 | 19 | 25.5 | 29 |
| 20 | 23 | 30 | 33 |
| 8 | 4 | 9.5 | 2.5 |
| 17 | 8 | 28 | 9.5 |
| 22 | 3 | 32 | 1 |
| 12 | 6 | 20 | 5 |
| 9 | 10 | 12 | 15 |
|  | 13 |  | 22 |
|  | 6 |  | 5 |
|  | 9 |  | 12 |
|  | 12 |  | 20 |
|  | 7 |  | 7.5 |
|  | 10 |  | 15 |
|  | 21 |  | 31 |
|  | 4 |  | 2.5 |
|  | 9 |  | 12 |
|  | 6 |  | 5 |
|  | 11 |  | 17.5 |
|  | 16 |  | 27 |
|  | 14 |  | 23.5 |
|  |  | = 221 | = 340 |

U1 = n1n2 + = 1023 + = 230 + = 64

U2 = n1n2 + = 1023 + = 230 + = 166

U0 = Minimum of (U1 or U2) = 64

Score 10 or rank 15 repeats 3 times between samples, t1 = 3, T1 = = = 2

Score 14 or rank 23.5 repeats 2 times between samples, t2 = 2, T2 = = = 0.5

Score 8 or rank 9.5 repeats 2 times between samples, t3 = 2, T3 = = = 0.5

Score 12 or rank 20 repeats 3 times between samples, t4 = 3, T4 = = = 2

Score 9 or rank 12 repeats 3 times between samples, t5 = 3, T5 = = = 2

ƩT = T1 + T2 + T3 + T4 + T5 = 2 + 0.5 + 0.5 + 2 + 2 + = 7

= = = 115

= = = = 25.4998

Now,

**Problem:** To test,

**Null hypothesis (H0):** F (X) = F(Y) ⇒(y); there is no significance difference between average rainfall of Dhanusha and Sarlahi.

**Alternative hypothesis H1:** F (X) < F(Y) ⇒; one-tailed test

**Test statistic:** Under H0, test statistics is given by

Z = = = - 2.00

**Critical region:** Next for a pre-assigned level of significance = 0.05, the probability (p0) is associated with the values as extreme as z, we obtained from z- table is

p0 = p () = p () = 0.0228

**Decision:** Since, p0<, we reject H0. It is significant.